Numeric Response Questions

3D Geometry

Q.1 If the line $\frac{x-3}{2}=\frac{y-4}{3}=\frac{z-5}{4}$ lies in the plane $4x+4y-kz-d=0$, then find the value of $(k+d)$.

Q.2 The distance of the point $(-1,-5,-10)$ from the point of intersection of line $\frac{x-2}{3}=\frac{y+1}{4}=\frac{z-2}{12}$ and plane $x-y+z=5$ is ' λ ' then find value of $[\lambda/2]$ (where [ ] representa greatest integer function)

Q.3 The plane passing through the point $(-2,-2,2)$ and containing the line joining the points $(1,1,1)$ and $(1,-1,2)$ makes intercepts on the coordinate axes, the sum of whose length is λ then find value of jz.

Q.4 The volume of the tetrahedron included between the plane $3x+4y-5z-60=0$ and the coordinate planes is λ then find value of $\frac{\lambda}{100}$.

Q.5 A plane is parallel to two lines whose direction ratios are $(1,0,-1)$ and $(-1,1,0)$ and it passes through the point $(1,1,1)$, cuts the axis at A, B, C, then find the volume of the tetrahedron $OABC$.

Q.6 If the reflection of the point $P(1,0,0)$ in the line $\frac{x-1}{2}=\frac{y+1}{-3}=\frac{z+10}{8}$ is (α,β,γ) then find value of $-(\alpha+\beta+\gamma)$

Q.7 A line makes the same angle θ, with each of the x and z axes. If the angle β, which it makes with y-axis is such that $\sin^2\beta=3\sin^2\theta$, then find value of $5\cos^2\theta$.

Q.8 If line joining $A(8,4,5)$ and $B(4,3,9)$ are parallel to plane $\vec{r},(\hat{\imath}+\lambda\hat{\jmath}+\hat{k})=5$ then find value of λ.

Q.9 A variable plane is at a distance 2 from origin O and meets co-ordinate axes at A, B and C. The centroid of tetrahedron OABC lies on $\frac{1}{x^2}+\frac{1}{y^2}+\frac{1}{z^2}=k^2$ then find value of k.

Q.10 Find the distance of the point $(1,2,3)$ from the line $\frac{x-6}{3}=\frac{y-7}{2}=\frac{z-7}{-2}$,

Q.11 If the sum of the squares of the distance of a point from the three coordinate axes is 36, if its distance from the origin is $k\sqrt{2}$ then find k.

Q.12 If the shortest distance between the lines $\frac{x-3}{3}=\frac{y-8}{-1}=\frac{z-3}{1}$ and $\frac{x+3}{-3}=\frac{y+7}{2}=\frac{z-6}{4}$ is $k\sqrt{30}$ then find k.

Q.13 The image of point $(1,2,-3)$ in the plane $3x-3y+10z-26=0$ is (α,β,γ) then find $(\alpha+\beta+\gamma)$.

Q.14 Find algebraic sum of the intercepts made by the plane $x+3y-4x+6=0$ on the axes.

Q.15 If the straight line $x=1+\mathrm{s}, \mathrm{y}=3-\lambda\mathrm{s}, \mathrm{z}=1+\lambda\mathrm{s}$ and $x=t/2, y=1+t, z=2-t$, with parameters 's' and 't' respectively, are coplanar, then find the value of λ.

ANSWER KEY

1. 8.00	**2.** 4.00	**3.** 6.00	**4.** 6.00	**5.** 4.50	**6.** 7.00	**7.** 3.00
8. 5.00	**9.** 2.00	**10.** 7.00	**11.** 3.00	**12.** 3.00	**13.** 10.00	**14.** –6.50
15. –2.00						

Hints & Solutions

1. $4(2) + 4(3) - k(4) = 0$

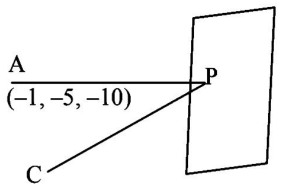
$k = 5$

(3, 4, 5) lies on plane

$4(3) + 4(4) - 5(5) - d = 0$

$d = 3$

2. Equation of plane $x - y + z = 5$ ... (1)



equation of line PC $\frac{x-2}{3} = \frac{y+1}{4} = \frac{z-2}{12} = r$

$x = 3r + 2,\ y = 4r - 1,\ z = 12r + 2$

coordinate of $P(3r + 2, 4r - 1, 12r + 2)$

Point P lies in plane (1)

$(3r + 2) - (4r - 1) + 12r + 2 = 5$

$11r + 5 = 5$

$r = 0$

coordinate of P(2, –1, 2)

$AP = \sqrt{9 + 16 + 144} = 13$

3. Any pane passing through the point (– 2,– 2, 2) is

$a(x + 2) + b(y + 2) + c\,(z - 2) = 0$ ... (i)

equation of line joining the given points

$\frac{x-1}{0} = \frac{y-1}{2} = \frac{z-1}{-1}$ ... (ii)

plane containing the line

so (1, 1, 1) lie on the plane and normal is perpendicular to the line which direction-ratio is (0, 2, –1)

$\therefore\ 3a + 3b - c = 0$

and $a.\,0 + 2b - c = 0$

intercept on the coordinate axis is 8, $\frac{8}{3}$, $\frac{8}{6}$ respectively

$\text{sum} = 8 + \frac{8}{3} + \frac{4}{3} = 12$

4. Equation of the given plane is

$\frac{x}{20} + \frac{y}{15} + \frac{z}{-12} = 1$

which meets the coordinate axes in points A(20, 0, 0), B(0, 15, 0), c(0, 0, –12)

and coordinate of the origine are (0, 0, 0)

volume of the tetrahedron OABC is

$$= \frac{1}{6}\begin{vmatrix} 0 & 0 & 0 & 1 \\ 20 & 0 & 0 & 1 \\ 0 & 15 & 0 & 1 \\ 0 & 0 & -12 & 1 \end{vmatrix}$$

$$= \left|\frac{1}{6} \times 20 \times 15 \times (-12)\right| = 600$$

5. Let equation of plane

$a(x - 1) + b(y - 1) + c(z - 1) = 0$

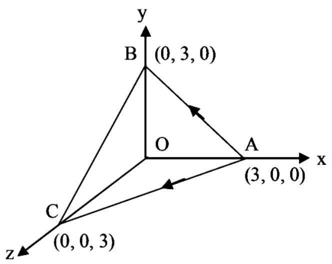
and $a - c = 0$ ...(1)

$-a + b = 0$

$a = b = c = \lambda$, from (1)

$\lambda(x - 1) + \lambda\,(y - 1) + \lambda(z - 1) = 0$

$x + y + z = 3$



$$\text{Volume} = \frac{1}{6}[\overrightarrow{OA}\ \ \overrightarrow{OB}\ \ \overrightarrow{OC}]$$

$$= \frac{1}{6}\begin{vmatrix} 3 & 0 & 0 \\ 0 & 3 & 0 \\ 0 & 0 & 3 \end{vmatrix} = \frac{3\times3\times3}{6} = \frac{9}{2}$$

6. Let reflection of P(1, 0, 0) in the line

$\frac{x-1}{2} = \frac{y+1}{-3} = \frac{z+10}{8}$ be (α, β, γ)

then $\left(\frac{\alpha+1}{2}, \frac{\beta}{2}, \frac{\gamma}{2}\right)$ lies on the line.

& $(\alpha-1)\,\hat{i} + \beta\hat{j} + \gamma\hat{k}$ is perpendicular

to $2\hat{i} - 3\hat{j} + 8\hat{k}$

$$\therefore \frac{\frac{\alpha+1}{2}-1}{2} = \frac{\frac{\beta}{2}+1}{-3} = \frac{\frac{\gamma}{2}+10}{8} = \lambda$$

and $2(\alpha - 1) - 3(\beta) + \gamma(8) = 0$

$\Rightarrow \alpha = 5, \beta = -8, \gamma = -4$

7. $\cos^2\theta + \cos^2\beta + \cos^2\theta = 1$

$\Rightarrow 2\cos^2\theta = \sin^2\beta$

but $\sin^2\beta = 3\sin^2\theta$

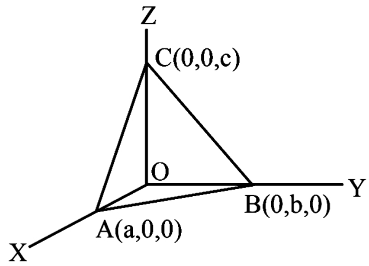
So, $3\sin^2\theta = 2\cos^2\theta$

$\Rightarrow 5\cos^2\theta = 3$

8. $\overrightarrow{AB} = \hat{i} - \hat{j} + 4\hat{k}$ must be perpendicular

to $\hat{i} + \lambda\hat{j} + \hat{k} \Rightarrow \lambda = 5$

9.



Let centroid of tetrahedron OABC $\equiv (\alpha, \beta, \gamma)$

$\frac{a}{4} = \alpha,\ \frac{b}{4} = \beta,\ \frac{c}{4} = \gamma$

$a = 4\alpha,\ b = 4\beta,\ c = 4\gamma$

equation of plane

$$\frac{x}{4\alpha} + \frac{y}{4\beta} + \frac{z}{4\gamma} = 1$$

distance from origin

$$= \frac{1}{\sqrt{\frac{1}{16\alpha^2} + \frac{1}{16\beta^2} + \frac{1}{16\gamma^2}}}$$

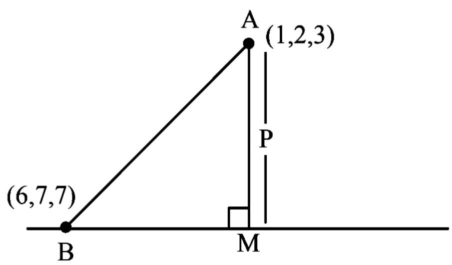
$$2 = \frac{1}{\sqrt{\frac{1}{16\alpha^2} + \frac{1}{16\beta^2} + \frac{1}{16\gamma^2}}}$$

$$\Rightarrow \frac{1}{16\alpha^2} + \frac{1}{16\beta^2} + \frac{1}{16\gamma^2} = \frac{1}{4}$$

$$\Rightarrow \frac{1}{\alpha^2} + \frac{1}{\beta^2} + \frac{1}{\gamma^2} = 4$$

So centroid lies on $\frac{1}{x^2} + \frac{1}{y^2} + \frac{1}{z^2} = 4$

10.



$$\frac{x-6}{3} = \frac{y-7}{2} = \frac{z-7}{-2}$$

$BM = \ell(x_2 - x_1) + m(y_2 - y_1) + n(z_2 - z_1)$

$$BM = \frac{3(6-1) + 2(7-2) + (-2)(7-3)}{\sqrt{17}}$$

$$BM = \frac{15+10-8}{\sqrt{17}} = \sqrt{17}$$

P = AM

$$= \sqrt{(AB)^2 - (BM)^2} = \sqrt{25+25+16-17}$$

P = 7

11. Let P(x, y, z) be the point. Now under given condition,

we get

$$[\sqrt{x^2+y^2}]^2 + [\sqrt{y^2+z^2}]^2 + [\sqrt{z^2+x^2}]^2$$

= 36

$\Rightarrow x^2 + y^2 + z^2 = 18$

Then distance from the origin to the point (x, y, z) is

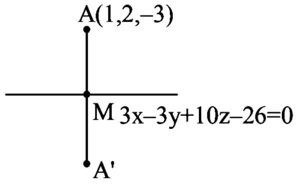
$$\sqrt{x^2+y^2+z^2} = \sqrt{18} = 3\sqrt{2}$$

12. Shortest distance = $\dfrac{\left\|\begin{matrix} 3+3 & 8+7 & 3-6 \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{matrix}\right\|}{\left\|\begin{matrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{matrix}\right\|}$

$$= \frac{\left\|\begin{matrix} 6 & 15 & -3 \\ 3 & -1 & 1 \\ -3 & 2 & 4 \end{matrix}\right\|}{|\hat{i}(-6) + \hat{j}(-15) + \hat{k}(3)|}$$

$$= \frac{|-36-225-9|}{\sqrt{36+225+9}} = \sqrt{270} = 3\sqrt{30}$$

13. Equation of line AM be



$$\frac{x-1}{3} = \frac{y-2}{-3} = \frac{z+3}{10}$$

Let M is $(3\lambda + 1, -3\lambda + 2, 10\lambda - 3)$

As M lies in plane

$9\lambda + 3 + 9\lambda - 6 + 100\lambda - 30 - 26 = 0$

$\Rightarrow 118\lambda = 59 \Rightarrow \lambda = \dfrac{1}{2}$

Hence M is (5/2, 1/2, 2)

Hence A' is (4, –1, 7)

14. $x + 3y - 4z + 6 = 0$

$$\Rightarrow \frac{x}{-6} + \frac{y}{-2} + \frac{z}{3/2} = 1$$

Algebraic sum of intercepts on axes

$= -6 - 2 + 3/2 = -13/2.$

15. Given lines can be written as

$$\frac{x-1}{1} = \frac{y-3}{-\lambda} = \frac{z-1}{\lambda} = s$$

$$\frac{x}{1/2} = \frac{y-1}{1} = \frac{z-2}{-1} = t$$

These lines are coplanar if

$$\begin{vmatrix} 1 & 2 & -1 \\ 1 & -\lambda & \lambda \\ 1/2 & 1 & -1 \end{vmatrix} = 0 \Rightarrow \lambda + 2 = 0 \Rightarrow \lambda = -2$$